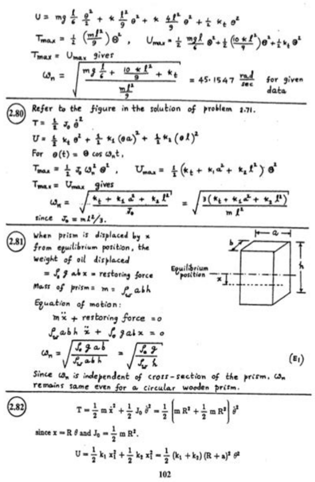

$$U = mg\frac{l}{6}\frac{\theta^2}{2} + k\frac{l^2}{9}\theta^2 + k\frac{4l^2}{9}\theta^2 + \frac{1}{2}k_t\theta^2$$

$$T_{max} = \frac{1}{2}\left(\frac{ml^2}{9}\right)\Theta^2, \quad U_{max} = \frac{1}{2}\frac{mgl}{6}\Theta^2 + \frac{1}{2}\left(\frac{10kl^2}{9}\right)\Theta^2 + \frac{1}{2}k_t\Theta^2$$

$T_{max} = U_{max}$ gives

$$\omega_n = \sqrt{\frac{mg\frac{l}{6} + \frac{10kl^2}{9} + k_t}{\frac{ml^2}{9}}} = 45.1547 \frac{rad}{sec} \text{ for given data}$$

(2.80) Refer to the figure in the solution of problem 2.71.

$$T = \frac{1}{2}J_0\dot{\theta}^2$$

$$U = \frac{1}{2}k_t\theta^2 + \frac{1}{2}k_1(\theta a)^2 + \frac{1}{2}k_2(\theta l)^2$$

For $\theta(t) = \Theta\cos\omega_n t$,

$$T_{max} = \frac{1}{2}J_0\omega_n^2\Theta^2, \quad U_{max} = \frac{1}{2}(k_t + k_1a^2 + k_2l^2)\Theta^2$$

$T_{max} = U_{max}$ gives

$$\omega_n = \sqrt{\frac{k_t + k_1a^2 + k_2l^2}{J_0}} = \sqrt{\frac{3(k_t + k_1a^2 + k_2l^2)}{ml^2}}$$

since $J_0 = ml^2/3$.

(2.81) When prism is displaced by x from equilibrium position, the weight of oil displaced $= \rho_o g\, ab\, x$ = restoring force

Mass of prism $= m = \rho_w abh$

Equation of motion:

$m\ddot{x}$ + restoring force $= 0$

$$\rho_w abh\,\ddot{x} + \rho_o g ab x = 0$$

$$\omega_n = \sqrt{\frac{\rho_o g ab}{\rho_w abh}} = \sqrt{\frac{\rho_o g}{\rho_w h}} \quad (E_1)$$

Since ω_n is independent of cross-section of the prism, ω_n remains same even for a circular wooden prism.

(2.82)

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}\left(mR^2 + \frac{1}{2}mR^2\right)\dot{\theta}^2$$

since $x = R\theta$ and $J_0 = \frac{1}{2}mR^2$.

$$U = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_1^2 = \frac{1}{2}(k_1 + k_2)(R + a)^2\theta^2$$

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