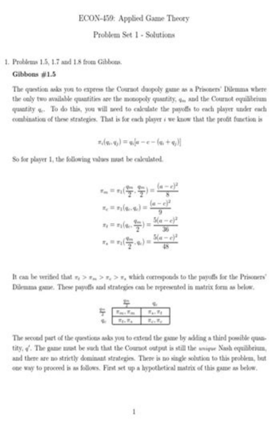

ECON-459: Applied Game Theory

Problem Set 1 - Solutions

1. Problems 1.5, 1.7 and 1.8 from Gibbons.

Gibbons #1.5

The question asks you to express the Cournot duopoly game as a Prisoners' Dilemma where the only two available quantities are the monopoly quantity, q_m and the Cournot equilibrium quantity q_c. To do this, you will need to calculate the payoffs to each player under each combination of these strategies. That is for each player i we know that the profit function is

$$\pi_i(q_i, q_j) = q_i[a - c - (q_i + q_j)]$$

So for player 1, the following values must be calculated.

$$\pi_m = \pi_1(\frac{q_m}{2}, \frac{q_m}{2}) = \frac{(a-c)^2}{8}$$
$$\pi_c = \pi_1(q_c, q_c) = \frac{(a-c)^2}{9}$$
$$\pi_t = \pi_1(q_c, \frac{q_m}{2}) = \frac{5(a-c)^2}{36}$$
$$\pi_s = \pi_1(\frac{q_m}{2}, q_c) = \frac{5(a-c)^2}{48}$$

It can be verified that $\pi_t > \pi_m > \pi_c > \pi_s$ which corresponds to the payoffs for the Prisoners' Dilemma game. These payoffs and strategies can be represented in matrix form as below.

	$\frac{q_m}{2}$	q_c
$\frac{q_m}{2}$	π_m, π_m	π_s, π_t
q_c	π_t, π_s	π_c, π_c

The second part of the questions asks you to extend the game by adding a third possible quantity, q'. The game must be such that the Cournot output is still the *unique* Nash equilibrium, and there are no strictly dominant strategies. There is no single solution to this problem, but one way to proceed is as follows. First set up a hypothetical matrix of this game as below.

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