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But X, G are independent, so : $p(x,y) = p(x)p(y)$, $p(x|y) = p(x)$.Hence :

$$\begin{aligned} H(X|G) &= -\int_{-\infty}^{\infty} p(y)\left[\int_{-\infty}^{\infty} p(x)\log p(x)dx\right]dy \\ &= -\int_{-\infty}^{\infty} p(y)H(X)dy \\ &= H(X) = \frac{1}{2}\log(2\pi e\sigma_x^2) \end{aligned}$$

where the last equality stems from the Gaussian pdf of X.

(b)

$$I(X;Y) = H(Y) - H(Y|X)$$

Since Y is the sum of two independent, zero-mean Gaussian r.v's , it is also a zero-mean Gaussian r.v. with variance : $\sigma_z^2 = \sigma_x^2 + \sigma_n^2$. Hence : $H(Y) = \frac{1}{2}\log\left(2\pi e\left(\sigma_x^2 + \sigma_n^2\right)\right)$. Also, since $y = x + g$:

$$p(y|x) = p_g(y - x) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(y-x)^2}{2\sigma_n^2}}$$

Hence :

$$\begin{aligned} H(Y|X) &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\log p(y|x)dxdy \\ &= -\int_{-\infty}^{\infty} p(x)\log e\int_{-\infty}^{\infty} p(y|x)\ln\left(\frac{1}{\sqrt{2\pi\sigma_n}}\exp\left(-\frac{(y-x)^2}{2\sigma_n^2}\right)\right)dydx \\ &= \int_{-\infty}^{\infty} p(x)\log e\left[\int_{-\infty}^{\infty} p_g(y-x)\left(\ln(\sqrt{2\pi}\sigma_n) + \frac{(y-x)^2}{2\sigma_n^2}\right)dy\right]dx \\ &= \int_{-\infty}^{\infty} p(x)\log e\left[\ln(\sqrt{2\pi}\sigma_n) + \frac{1}{2\sigma_n^2}\sigma_n^2\right]dx \\ &= \left[\log(\sqrt{2\pi}\sigma_n) + \frac{1}{2}\log e\right]\int_{-\infty}^{\infty} p(x)dx \\ &= \frac{1}{2}\log\left(2\pi e\sigma_n^2\right) \quad (= H(G)) \end{aligned}$$

where we have used the fact that : $\int_{-\infty}^{\infty} p_g(y-x)dy = 1$, $\int_{-\infty}^{\infty}(y-x)^2 p_g(y-x)dy = E[G^2] = \sigma_n^2$.
From $H(Y)$, $H(Y|X)$:

$$I(X;Y) = H(Y) - H(Y|X) = \frac{1}{2}\log\left(2\pi e(\sigma_x^2 + \sigma_n^2)\right) - \frac{1}{2}\log\left(2\pi e\sigma_n^2\right) = \frac{1}{2}\log\left(1 + \frac{\sigma_x^2}{\sigma_n^2}\right)$$

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