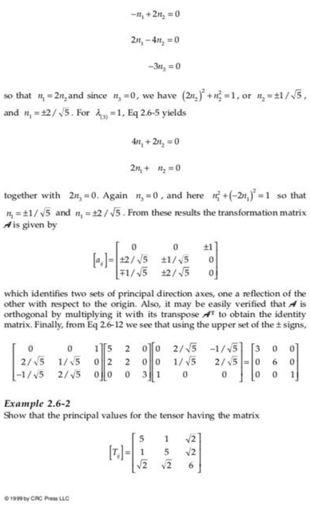

$$-n_1 + 2n_2 = 0$$

$$2n_1 - 4n_2 = 0$$

$$-3n_3 = 0$$

so that $n_1 = 2n_2$ and since $n_3 = 0$, we have $(2n_2)^2 + n_2^2 = 1$, or $n_2 = \pm 1/\sqrt{5}$, and $n_1 = \pm 2/\sqrt{5}$. For $\lambda_{(3)} = 1$, Eq 2.6-5 yields

$$4n_1 + 2n_2 = 0$$

$$2n_1 + n_2 = 0$$

together with $2n_3 = 0$. Again $n_3 = 0$, and here $n_1^2 + (-2n_1)^2 = 1$ so that $n_1 = \pm 1/\sqrt{5}$ and $n_1 = \pm 2/\sqrt{5}$. From these results the transformation matrix $\mathcal{A}$ is given by

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & \pm 1 \\ \pm 2/\sqrt{5} & \pm 1/\sqrt{5} & 0 \\ \mp 1/\sqrt{5} & \pm 2/\sqrt{5} & 0 \end{bmatrix}$$

which identifies two sets of principal direction axes, one a reflection of the other with respect to the origin. Also, it may be easily verified that $\mathcal{A}$ is orthogonal by multiplying it with its transpose $\mathcal{A}^T$ to obtain the identity matrix. Finally, from Eq 2.6-12 we see that using the upper set of the ± signs,

$$\begin{bmatrix} 0 & 0 & 1 \\ 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \end{bmatrix} \begin{bmatrix} 5 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2/\sqrt{5} & -1/\sqrt{5} \\ 0 & 1/\sqrt{5} & 2/\sqrt{5} \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 2.6-2
Show that the principal values for the tensor having the matrix

$$[T_{ij}] = \begin{bmatrix} 5 & 1 & \sqrt{2} \\ 1 & 5 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 6 \end{bmatrix}$$