6.1. a. $E_t = E_1 + E_2 + E_3 = 2E_0 \frac{e^{-jkr}}{r} + E_0 \frac{e^{-jkr_1}}{r_1} + E_0 \frac{e^{-jkr_2}}{r_2}$

where the center element is placed at the origin. For far-field observations

$r_1 \simeq r - d\cos\theta$, $r_2 \simeq r + d\cos\theta$ } for phase variations

$r_1 \simeq r_2 \simeq r$ for amplitude variations

and

$E_t \simeq E_0 \frac{e^{-jkr}}{r}\{2 + e^{jkd\cos\theta} + e^{-jkd\cos\theta}\}$

$\simeq E_0 \frac{e^{-jkr}}{r}\{2[1 + \frac{1}{2}(e^{jkd\cos\theta} + e^{-jkd\cos\theta})]\}$

$= E_0 \frac{e^{-jkr}}{r}\{2[1 + \cos(kd\cos\theta)]\}$

Computer Program
Directivity
$U = \cos^4(\frac{\pi}{4}\cos\theta)$
At. $d = \lambda/4$
$P_{rad} = 8.7119$
$D_0 = 1.44244$
$= 1.5910$ dB
$kd = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$
$AF(\theta) = 4\cos^2(\frac{\pi}{4}\cos\theta)$

Thus the array factor is equal to

$AF(\theta) = 2[1 + \cos(kd\cos\theta)] = 4\cos^2(\frac{kd}{2}\cos\theta)$

which in normalized form can also be written as

$AF(\theta)_n = 1 + \cos(kd\cos\theta) = 2\cos^2(\frac{kd}{2}\cos\theta)$

b. The nulls of the pattern can be found using either of the above forms for the array factor. For example

One form	the other Form
$AF(\theta) = 1 + \cos(kd\cos\theta_n) = 0$	$2\cos^2(\frac{kd}{2}\cos\theta_n) = 0$
$\cos(kd\cos\theta_n) = -1$	$\frac{kd}{2}\cos\theta_n = \cos^{-1}(0) = \frac{n\pi}{2}$, $n = \pm1, \pm3, \cdots$
$kd\cos\theta_n = \cos^{-1}(-1) = n\pi$, $n = \pm1, \pm3, \cdots$	$\theta_n = \cos^{-1}(n\lambda/(2d))$, $n = \pm1, \pm3, \cdots$
$\theta_n = \cos^{-1}(n\lambda/2d)$, $n = \pm1, \pm3, \pm5, \cdots$	

which are of identical form. Therefore both forms yield the same results. Thus for $d = \lambda/4$

$\theta_n = \cos^{-1}(\frac{n\lambda}{2d})\Big|_{d=\lambda/4} = \cos^{-1}(2n)$, $n = \pm1, \pm3, \cdots \Rightarrow$ No nulls exist.

c. Similarly the maxima of the pattern can be found using either of the two forms for the array factor. For example

(Continued)

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