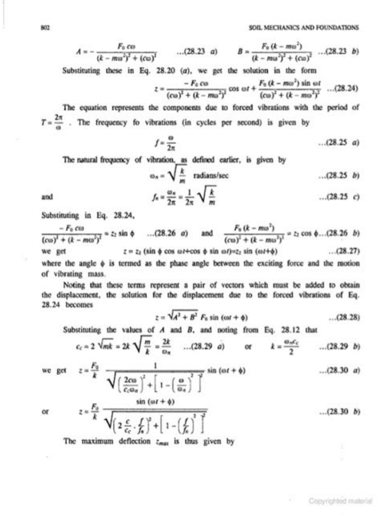

802 SOIL MECHANICS AND FOUNDATIONS

$$A = -\frac{F_0 c\omega}{(k - m\omega^2)^2 + (c\omega)^2} \quad ...(28.23\ a) \qquad B = \frac{F_0 (k - m\omega^2)}{(k - m\omega^2)^2 + (c\omega)^2} \quad ...(28.23\ b)$$

Substituting these in Eq. 28.20 (*a*), we get the solution in the form

$$z = \frac{-F_0 c\omega}{(c\omega)^2 + (k - m\omega^2)^2} \cos \omega t + \frac{F_0 (k - m\omega^2) \sin \omega t}{(c\omega)^2 + (k - m\omega^2)^2} \quad ...(28.24)$$

The equation represents the components due to forced vibrations with the period of $T = \frac{2\pi}{\omega}$. The frequency fo vibrations (in cycles per second) is given by

$$f = \frac{\omega}{2\pi} \quad ...(28.25\ a)$$

The natural frequency of vibration, as defined earlier, is given by

$$\omega_n = \sqrt{\frac{k}{m}} \text{ radians/sec} \quad ...(28.25\ b)$$

and

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad ...(28.25\ c)$$

Substituting in Eq. 28.24,

$$\frac{-F_0 c\omega}{(c\omega)^2 + (k - m\omega^2)^2} = z_1 \sin \phi \quad ...(28.26\ a) \quad \text{and} \quad \frac{F_0 (k - m\omega^2)}{(c\omega)^2 + (k - m\omega^2)^2} = z_1 \cos \phi \quad ...(28.26\ b)$$

we get

$$z = z_1 (\sin \phi \cos \omega t + \cos \phi \sin \omega t) = z_1 \sin (\omega t + \phi) \quad ...(28.27)$$

where the angle $\phi$ is termed as the phase angle between the exciting force and the motion of vibrating mass.

Noting that these terms represent a pair of vectors which must be added to obtain the displacement, the solution for the displacement due to the forced vibrations of Eq. 28.24 becomes

$$z = \sqrt{A^2 + B^2}\, F_0 \sin (\omega t + \phi) \quad ...(28.28)$$

Substituting the values of $A$ and $B$, and noting from Eq. 28.12 that

$$c_c = 2\sqrt{mk} = 2k\sqrt{\frac{m}{k}} = \frac{2k}{\omega_n} \quad ...(28.29\ a) \qquad \text{or} \qquad k = \frac{\omega_n c_c}{2} \quad ...(28.29\ b)$$

we get

$$z = \frac{F_0}{k} \frac{1}{\sqrt{\left(\frac{2c\omega}{c_c \omega_n}\right)^2 + \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2}} \sin (\omega t + \phi) \quad ...(28.30\ a)$$

or

$$z = \frac{F_0}{k} \frac{\sin (\omega t + \phi)}{\sqrt{\left(2\frac{c}{c_c} \cdot \frac{f}{f_n}\right)^2 + \left[1 - \left(\frac{f}{f_n}\right)^2\right]^2}} \quad ...(28.30\ b)$$

The maximum deflection $z_{max}$ is thus given by

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