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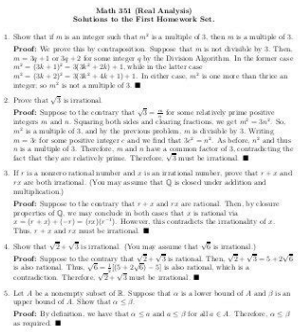

**Math 351 (Real Analysis)**
**Solutions to the First Homework Set.**

1. Show that if $m$ is an integer such that $m^2$ is a multiple of 3, then $m$ is a multiple of 3.

   **Proof:** We prove this by contraposition. Suppose that $m$ is not divisible by 3. Then, $m = 3q + 1$ or $3q + 2$ for some integer $q$ by the Division Algorithm. In the former case $m^2 = (3k+1)^2 = 3(3k^2+2k)+1$, while in the latter case $m^2 = (3k+2)^2 = 3(3k^2+4k+1)+1$. In either case, $m^2$ is one more than thrice an integer; so $m^2$ is not a multiple of 3. ■

2. Prove that $\sqrt{3}$ is irrational.

   **Proof:** Suppose to the contrary that $\sqrt{3} = \frac{m}{n}$ for some relatively prime positive integers $m$ and $n$. Squaring both sides and clearing fractions, we get $m^2 = 3n^2$. So, $m^2$ is a multiple of 3, and by the previous problem, $m$ is divisible by 3. Writing $m = 3c$ for some positive integer $c$ and we find that $3c^2 = n^2$. As before, $n^2$ and thus $n$ is a multiple of 3. Therefore, $m$ and $n$ have a common factor of 3, contradicting the fact that they are relatively prime. Therefore, $\sqrt{3}$ must be irrational. ■

3. If $r$ is a nonzero rational number and $x$ is an irrational number, prove that $r + x$ and $rx$ are both irrational. (You may assume that $\mathbb{Q}$ is closed under addition and multiplication.)

   **Proof:** Suppose to the contrary that $r + x$ and $rx$ are rational. Then, by closure properties of $\mathbb{Q}$, we may conclude in both cases that $x$ is rational via $x = (r+x) + (-r) = (rx)(r^{-1})$. However, this contradicts the irrationality of $x$. Thus, $r + x$ and $rx$ must be irrational. ■

4. Show that $\sqrt{2} + \sqrt{3}$ is irrational. (You may assume that $\sqrt{6}$ is irrational.)

   **Proof:** Suppose to the contrary that $\sqrt{2} + \sqrt{3}$ is rational. Then, $\sqrt{2} + \sqrt{3} = 5 + 2\sqrt{6}$ is also rational. Thus, $\sqrt{6} = \frac{1}{2}[(5 + 2\sqrt{6}) - 5]$ is also rational, which is a contradiction. Therefore, $\sqrt{2} + \sqrt{3}$ must be irrational. ■

5. Let $A$ be a nonempty subset of $\mathbb{R}$. Suppose that $\alpha$ is a lower bound of $A$ and $\beta$ is an upper bound of $A$. Show that $\alpha \leq \beta$.

   **Proof:** By definition, we have that $\alpha \leq a$ and $a \leq \beta$ for all $a \in A$. Therefore, $\alpha \leq \beta$ as required. ■

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