Suppose $1 \le p < \infty$. If $\int |f_n - f|^p \, d\mu \to 0$, then $f_n \to f$ in measure, and hence some subsequence converges to f a.e. On the other hand, if $f_n \to f$ in measure and $|f_n| \le g \in L^p$ for all n, then $\int |f_n - f|^p \, d\mu \to 0$.

Proof: Define the set $E_{n,\epsilon} := \{x : |f_n(x) - f(x)|^p \ge \epsilon^p\}$. Then

$$\int |f_n - f|^p \, d\mu \ge \int_{E_{n,\epsilon}} |f_n - f|^p \, d\mu \ge \int_{E_{n,\epsilon}} \epsilon^p \, d\mu = \epsilon^p \mu(E_{n,\epsilon}),$$

and thus as $n \to \infty$, we have that

$$\mu(E_{n,\epsilon}) \le \frac{1}{\epsilon^p} \int |f_n - f|^p \, d\mu \to 0.$$

Hence, $\mu(E_{n,\epsilon}) \to 0$. Thus, $f_n \to f$ in measure.