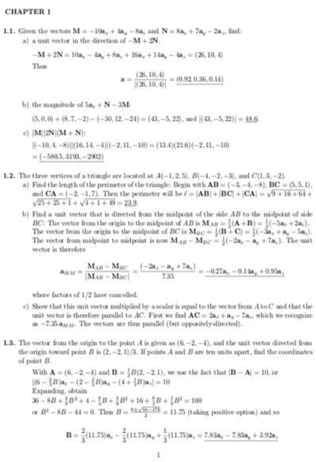

CHAPTER 1

1.1. Given the vectors $\mathbf{M} = -10\mathbf{a}_x + 4\mathbf{a}_y - 8\mathbf{a}_z$ and $\mathbf{N} = 8\mathbf{a}_x + 7\mathbf{a}_y - 2\mathbf{a}_z$, find:

a) a unit vector in the direction of $-\mathbf{M} + 2\mathbf{N}$.

$-\mathbf{M} + 2\mathbf{N} = 10\mathbf{a}_x - 4\mathbf{a}_y + 8\mathbf{a}_z + 16\mathbf{a}_x + 14\mathbf{a}_y - 4\mathbf{a}_z = (26, 10, 4)$

Thus

$$\mathbf{a} = \frac{(26,10,4)}{|(26,10,4)|} = \underline{(0.92, 0.36, 0.14)}$$

b) the magnitude of $5\mathbf{a}_x + \mathbf{N} - 3\mathbf{M}$:

$(5,0,0) + (8,7,-2) - (-30,12,-24) = (43,-5,22)$, and $|(43,-5,22)| = \underline{48.6}$

c) $|\mathbf{M}||2\mathbf{N}|(\mathbf{M} + \mathbf{N})$:

$|(-10,4,-8)||(16,14,-4)|(-2,11,-10) = (13.4)(21.6)(-2,11,-10)$

$= \underline{(-580.5, 3193, -2902)}$

1.2. The three vertices of a triangle are located at $A(-1,2,5)$, $B(-4,-2,-3)$, and $C(1,3,-2)$.

a) Find the length of the perimeter of the triangle: Begin with $\mathbf{AB} = (-3,-4,-8)$, $\mathbf{BC} = (5,5,1)$, and $\mathbf{CA} = (-2,-1,7)$. Then the perimeter will be $\ell = |\mathbf{AB}| + |\mathbf{BC}| + |\mathbf{CA}| = \sqrt{9+16+64} + \sqrt{25+25+1} + \sqrt{4+1+49} = \underline{23.9}$.

b) Find a unit vector that is directed from the midpoint of the side AB to the midpoint of side BC: The vector from the origin to the midpoint of AB is $\mathbf{M}_{AB} = \frac{1}{2}(\mathbf{A}+\mathbf{B}) = \frac{1}{2}(-5\mathbf{a}_x + 2\mathbf{a}_z)$. The vector from the origin to the midpoint of BC is $\mathbf{M}_{BC} = \frac{1}{2}(\mathbf{B}+\mathbf{C}) = \frac{1}{2}(-3\mathbf{a}_x + \mathbf{a}_y - 5\mathbf{a}_z)$. The vector from midpoint to midpoint is now $\mathbf{M}_{AB} - \mathbf{M}_{BC} = \frac{1}{2}(-2\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z)$. The unit vector is therefore

$$\mathbf{a}_{MM} = \frac{\mathbf{M}_{AB} - \mathbf{M}_{BC}}{|\mathbf{M}_{AB} - \mathbf{M}_{BC}|} = \frac{(-2\mathbf{a}_x - \mathbf{a}_y + 7\mathbf{a}_z)}{7.35} = \underline{-0.27\mathbf{a}_x - 0.14\mathbf{a}_y + 0.95\mathbf{a}_z}$$

where factors of 1/2 have cancelled.

c) Show that this unit vector multiplied by a scalar is equal to the vector from A to C and that the unit vector is therefore parallel to AC. First we find $\mathbf{AC} = 2\mathbf{a}_x + \mathbf{a}_y - 7\mathbf{a}_z$, which we recognize as $-7.35\,\mathbf{a}_{MM}$. The vectors are thus parallel (but oppositely-directed).

1.3. The vector from the origin to the point A is given as $(6,-2,-4)$, and the unit vector directed from the origin toward point B is $(2,-2,1)/3$. If points A and B are ten units apart, find the coordinates of point B.

With $\mathbf{A} = (6,-2,-4)$ and $\mathbf{B} = \frac{1}{3}B(2,-2,1)$, we use the fact that $|\mathbf{B} - \mathbf{A}| = 10$, or

$|(6 - \frac{2}{3}B)\mathbf{a}_x - (2 - \frac{2}{3}B)\mathbf{a}_y - (4 + \frac{1}{3}B)\mathbf{a}_z| = 10$

Expanding, obtain

$36 - 8B + \frac{4}{9}B^2 + 4 - \frac{8}{3}B + \frac{4}{9}B^2 + 16 + \frac{8}{3}B + \frac{1}{9}B^2 = 100$

or $B^2 - 8B - 44 = 0$. Thus $B = \frac{8 \pm \sqrt{64-176}}{2} = 11.75$ (taking positive option) and so

$$\mathbf{B} = \frac{2}{3}(11.75)\mathbf{a}_x - \frac{2}{3}(11.75)\mathbf{a}_y + \frac{1}{3}(11.75)\mathbf{a}_z = \underline{7.83\mathbf{a}_x - 7.83\mathbf{a}_y + 3.92\mathbf{a}_z}$$

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