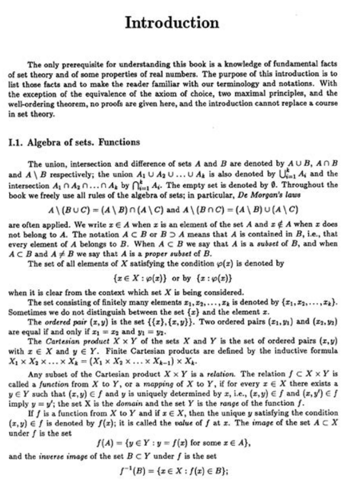

Introduction

The only prerequisite for understanding this book is a knowledge of fundamental facts of set theory and of some properties of real numbers. The purpose of this introduction is to list those facts and to make the reader familiar with our terminology and notations. With the exception of the equivalence of the axiom of choice, two maximal principles, and the well-ordering theorem, no proofs are given here, and the introduction cannot replace a course in set theory.

I.1. Algebra of sets. Functions

The union, intersection and difference of sets A and B are denoted by $A \cup B$, $A \cap B$ and $A \setminus B$ respectively; the union $A_1 \cup A_2 \cup \ldots \cup A_k$ is also denoted by $\bigcup_{i=1}^{k} A_i$ and the intersection $A_1 \cap A_2 \cap \ldots \cap A_k$ by $\bigcap_{i=1}^{k} A_i$. The empty set is denoted by $\emptyset$. Throughout the book we freely use all rules of the algebra of sets; in particular, *De Morgan's laws*

$$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C) \text{ and } A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

are often applied. We write $x \in A$ when x is an element of the set A and $x \notin A$ when x does not belong to A. The notation $A \subset B$ or $B \supset A$ means that A is contained in B, i.e., that every element of A belongs to B. When $A \subset B$ we say that A is a *subset* of B, and when $A \subset B$ and $A \neq B$ we say that A is a *proper subset* of B.

The set of all elements of X satisfying the condition $\varphi(x)$ is denoted by

$$\{x \in X : \varphi(x)\} \text{ or by } \{x : \varphi(x)\}$$

when it is clear from the context which set X is being considered.

The set consisting of finitely many elements $x_1, x_2, \ldots, x_k$ is denoted by $\{x_1, x_2, \ldots, x_k\}$. Sometimes we do not distinguish between the set $\{x\}$ and the element x.

The *ordered pair* (x, y) is the set $\{\{x\}, \{x, y\}\}$. Two ordered pairs (x_1, y_1) and (x_2, y_2) are equal if and only if $x_1 = x_2$ and $y_1 = y_2$.

The *Cartesian product* $X \times Y$ of the sets X and Y is the set of ordered pairs (x, y) with $x \in X$ and $y \in Y$. Finite Cartesian products are defined by the inductive formula $X_1 \times X_2 \times \ldots \times X_k = (X_1 \times X_2 \times \ldots \times X_{k-1}) \times X_k$.

Any subset of the Cartesian product $X \times Y$ is a *relation*. The relation $f \subset X \times Y$ is called a *function* from X to Y, or a *mapping* of X to Y, if for every $x \in X$ there exists a $y \in Y$ such that $(x, y) \in f$ and y is uniquely determined by x, i.e., $(x, y) \in f$ and $(x, y') \in f$ imply $y = y'$; the set X is the *domain* and the set Y is the *range* of the function f.

If f is a function from X to Y and if $x \in X$, then the unique y satisfying the condition $(x, y) \in f$ is denoted by $f(x)$; it is called the *value* of f at x. The *image* of the set $A \subset X$ under f is the set

$$f(A) = \{y \in Y : y = f(x) \text{ for some } x \in A\},$$

and the *inverse image* of the set $B \subset Y$ under f is the set

$$f^{-1}(B) = \{x \in X : f(x) \in B\};$$