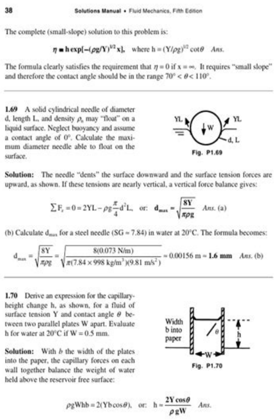

38 **Solutions Manual** • Fluid Mechanics, Fifth Edition

The complete (small-slope) solution to this problem is:

$$\eta = h\exp[-(\rho g/Y)^{1/2}x], \quad \text{where } h = (Y/\rho g)^{1/2}\cot\theta \quad \textit{Ans.}$$

The formula clearly satisfies the requirement that $\eta = 0$ if $x = \infty$. It requires "small slope" and therefore the contact angle should be in the range $70° < \theta < 110°$.

1.69 A solid cylindrical needle of diameter d, length L, and density ρ_n may "float" on a liquid surface. Neglect buoyancy and assume a contact angle of 0°. Calculate the maximum diameter needle able to float on the surface.

Fig. P1.69

Solution: The needle "dents" the surface downward and the surface tension forces are upward, as shown. If these tensions are nearly vertical, a vertical force balance gives:

$$\sum F_z = 0 = 2YL - \rho g\frac{\pi}{4}d^2L, \quad \text{or:} \quad \mathbf{d_{max}} = \sqrt{\frac{8Y}{\pi\rho g}} \quad \textit{Ans. (a)}$$

(b) Calculate d_{max} for a steel needle (SG = 7.84) in water at 20°C. The formula becomes:

$$d_{max} = \sqrt{\frac{8Y}{\pi\rho g}} = \sqrt{\frac{8(0.073\ \text{N/m})}{\pi(7.84 \times 998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}} \approx 0.00156\ \text{m} \approx \mathbf{1.6\ mm} \quad \textit{Ans. (b)}$$

1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20°C if W = 0.5 mm.

Fig. P1.70

Solution: With b the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho gWhb = 2(Yb\cos\theta), \quad \text{or:} \quad h \approx \frac{2Y\cos\theta}{\rho gW} \quad \textit{Ans.}$$